

Full length article

Effect of complex damping on seismic responses of a reticulated dome and shaking table test validation

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ABSTRACT

The dynamic responses of large-span space structures are significantly associated with some higher structural frequency modes. A proper finite element model with accurate modal damping ratios is a key factor in obtaining precise structural seismic responses in numerical analyses. The complex damping model is often used in frequency domain structural analyses because of its characteristics of constant modal damping ratio with frequency and frequency-independent energy dissipation. Based on the five-parameter integer-order derivative method, a user subroutine of ABAQUS is developed to estimate the complex damping model in time-domain dynamic analyses. A shaking table test of a scaled and simplified single-layer reticulated dome is first conducted to validate the user subroutine. Detailed test data, including the identified natural frequencies and modal damping ratios, free vibration curves, and representative maximum responses, are summarized and compared with the corresponding finite element results using the complex damping model and a Rayleigh damping model. The comparisons indicate that the results of the complex damping model can yield a better estimation of the test data. Finally, the effect of the complex damping on the seismic responses of a single-layer reticulated dome is systematically studied by comparing the structural frequency properties and dynamic characteristic responses with those of Rayleigh damping. The results show that the complex damping model provides smaller structural modal damping ratio variation, lower ultimate seismic loading, and severer structural plastic developments.

1. Introduction

In the last 30 years, increasing number of large-span space structures have been constructed as grand landmarks in Chinese cities. Some of these public buildings were built in earthquake-prone areas, which attracts researchers' interest in the dynamic performance of large-span space structures. Owing to the fabrication complexities of test models and high experimental costs, most researchers used the finite element method to determine the dynamic responses of reticulated domes. Kato et al. conducted dynamic buckling analyses of a single-layer reticulated dome during earthquakes [1,2]. Zhi et al. systematically studied two possible failure mechanisms of a single-layer reticulated dome under severe earthquakes [3]. With the development of computational techniques, researchers began to study more complicated effects of the reticulated dome in practice [4–8].

Most of the literature above used the Rayleigh damping model to conduct dynamic analyses because the Rayleigh damping model is simple and computationally efficient in finite element analyses. In

general, Rayleigh damping is suitable for estimating the dynamic responses of some conventional structures like RC frames since the first few modes of these buildings significantly contribute to the structural dynamic responses [9]. However, Rayleigh damping will lead to some unreasonable physical responses such as frequency-dependent energy dissipation. Makris and Zhang showed that Rayleigh damping over-damped the higher vibration modes and yielded inappropriate approximation results for structural dynamic responses in some cases [10]. Additionally, if the overall space structure incorporates different materials, it will introduce inconveniences and difficulties when determining the coefficients of the Rayleigh damping model.

The contributions of some higher natural vibration modes of reticulated domes cannot be neglected [11–13]. The idea of introducing a complex damping model into a finite element model was proposed. Theodorsen and Garrick first proposed a linear structural friction model, also called a constant hysteretic model or complex damping model, in frequency domain analysis [14]. This model could guarantee that the energy losses per cycle were independent of structural

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frequencies that were observed in material experiments and building observations [15–17]. In a time domain analysis, Biot [18] and Caughey [19] developed the Voigt model of viscoelasticity and an exponential kernel function to simulate a constant hysteretic model over a wide structural frequency range. Biot's model was recently referred to as the Maxwell–Wiechert model [20].

The standard linear viscoelastic model, named the Kelvin–Maxwell model, can be also expressed as a constitutive equation with a series of integer-order time derivatives of stress and strain [21]. Liao used this integer-order derivative equation with five material constants to approximate a complex damping model within a practice-focused frequency range [22]. This five-parameter integer-order derivative model proved to be simple, causal, approximately frequency independent, and feasible when used in commercial finite element software to conduct time-domain analyses.

In this paper, the five-parameter integer-order derivative model is utilized to take into account the complex damping model in time-domain dynamic analyses of a single-layer reticulated dome under earthquakes. Because the five-parameter integer-order derivative model is not strictly equal to the ideal complex damping model, some dynamic test results must be used to validate this method. Some previous shaking table tests on single-layer reticulated domes [23–25] have been conducted, but most tests focused on phenomena descriptions and detailed data were not provided. Therefore, in the first section, a shake table test is carried out to obtain detailed data, especially the structural free vibration data, to verify the application of the complex damping model in a reticulated dome. In the next section, the differences in the seismic responses of a 40-m-span single-layer reticulated dome with the complex damping model and a Rayleigh damping model are thoroughly investigated.

2. Five-parameter integer-order derivative model

The Rayleigh damping (RD) model is simple and popular in structural time-domain transient dynamic analyses. The damping matrix $[C]$ assumes a linear combination of the mass matrix $[M]$ and stiffness matrix $[K]$:

$$[C] = \alpha_M [M] + \beta_K [K] \quad (1)$$

where α_M is the mass-proportional damping coefficient and β_K is the stiffness-proportional damping coefficient.

Unlike Rayleigh damping, complex damping (CD) is widely used in frequency-domain analyses. For an N-DoF system in frequency domain, the complex stiffness term, combined with stiffness and complex damping, can be presented as:

$$[G] = [1 + i\eta(\omega)\text{sign}(\omega)][K] \quad (2)$$

where $i = \sqrt{-1}$, ω is the frequency variable, $\eta(\omega)$ is the loss factor, and $\text{sign}(\cdot)$ is a function to get the sign of the variable.

Because the complex damping model is non-causal [16,19], the complex damping model can not be directly applied to ABAQUS in real-value time-domain transient dynamic analyses. To overcome the non-causality and complexity of time-domain analyses, an integer-order derivative constitutive equation was proposed to approximate the complex damping in level of material constitutive law [21]. The general form of the integer-order derivative constitutive equation is

$$\sigma + \sum_{m=1}^M p_m \frac{d^m \sigma}{dt^m} = q_0 \varepsilon + \sum_{n=1}^N q_n \frac{d^n \varepsilon}{dt^n} \quad (3)$$

where t is the time, σ is the stress, ε is the strain, and p_m , q_0 , and q_n are material constants. It is noted that the five-material-constants model is sufficiently accurate to approximate the complex damping model with a loss factor η_0 lower than 16% or damping ratio ξ_0 lower than 8% in the frequency range $[r_\omega \omega_H, \omega_H]$. ω_H is the highest interested frequency, and r_ω is the ratio between the lowest interested frequency and the highest,

usually set to 1/20 [22]. The principle to set the value for ω_H is to ensure that all frequencies of interested structural modes are included in the range $[r_\omega \omega_H, \omega_H]$.

Eq. (3) then becomes

$$\sigma + \frac{1}{\omega_H} \bar{p}_1 \frac{d\sigma}{dt} + \frac{1}{\omega_H^2} \bar{p}_2 \frac{d^2 \sigma}{dt^2} = E_0 \bar{q}_0 \varepsilon + \frac{E_0}{\omega_H} \bar{q}_1 \frac{d\varepsilon}{dt} + \frac{E_0}{\omega_H^2} \bar{q}_2 \frac{d^2 \varepsilon}{dt^2} \quad (4)$$

where E_0 is the material Young's modulus, and $\bar{p}_1$, $\bar{p}_2$, $\bar{q}_0$, $\bar{q}_1$, and $\bar{q}_2$ are the dimensionless material constants. It is noteworthy that $\bar{p}_1$ and $\bar{p}_2$ must be greater than 0.0 to ensure stress σ in Eq. (4) converges with the increasing of time t .

Liao proposed the following procedures to calculate the dimensionless material constants in Eq. (4) [22]. After utilizing Fourier transform to each side of Eq. (4), the equation can be summarized as

$$T(\omega) = E_0 \frac{\bar{q}_0 + \bar{q}_1 \frac{\omega}{\omega_H} i - \bar{q}_2 \left(\frac{\omega}{\omega_H}\right)^2}{1 + \bar{p}_1 \frac{\omega}{\omega_H} i - \bar{p}_2 \left(\frac{\omega}{\omega_H}\right)^2} \Gamma(\omega) \quad (5)$$

where $T(\omega)$ and $\Gamma(\omega)$ are the Fourier transform of the stress and strain histories respectively.

According to the definition of complex damping in frequency domain, the relationship between the stress and strain is expressed as

$$T(\omega) = [1 + \eta(\omega)i]E(\omega)\Gamma(\omega) \quad (6)$$

By comparing Eqs. (5) and (6), $\eta(\omega)$ and $E(\omega)$ can be solved as

$$\eta(\omega) = \frac{(\bar{q}_1 - \bar{p}_1 \bar{q}_0) \frac{\omega}{\omega_H} + (\bar{p}_1 \bar{q}_2 - \bar{p}_2 \bar{q}_1) \left(\frac{\omega}{\omega_H}\right)^3}{\bar{q}_0 + (\bar{p}_1 \bar{q}_1 - \bar{p}_2 \bar{q}_0 - \bar{q}_2) \left(\frac{\omega}{\omega_H}\right)^2 + \bar{p}_2 \bar{q}_2 \left(\frac{\omega}{\omega_H}\right)^4} \quad (7)$$

and

$$E(\omega) = E_0 \frac{\bar{q}_0 + (\bar{p}_1 \bar{q}_1 - \bar{p}_2 \bar{q}_0 - \bar{q}_2) \left(\frac{\omega}{\omega_H}\right)^2 + \bar{p}_2 \bar{q}_2 \left(\frac{\omega}{\omega_H}\right)^4}{1 + (\bar{p}_1^2 - 2\bar{p}_2) \left(\frac{\omega}{\omega_H}\right)^2 + \bar{p}_2^2 \left(\frac{\omega}{\omega_H}\right)^4} \quad (8)$$

From Eqs. (7) and (8), damping ratio η and material stiffness E vary with frequency ω instead of retaining constants. Liao [22] introduced the following objective function in frequency range $[r_\omega \omega_H, \omega_H]$

$$\psi = \int_{r_\omega \omega_H}^{\omega_H} \left[\left(\frac{E(\omega) - E_0}{E_0} \right)^2 + \left(\frac{\eta(\omega) - \eta_0}{\eta_0} \right)^2 \right] d\omega \quad (9)$$

in which less variations of E and η around the desired E_0 and η_0 will result in smaller ψ .

In order to obtain corresponding minimum ψ for different damping ratios ξ_0 , optimization method of Sequential Weight Increasing Factor Technique (SWIFT) [26] is adopted to calculate $\bar{p}_1$, $\bar{p}_2$, $\bar{q}_0$, $\bar{q}_1$, and $\bar{q}_2$. These five dimensionless material constants under different damping ratios ξ_0 are listed in Appendix Table A1. Appendix Fig. A1 presents the curves of dimensionless modal damping ratio ξ/ξ_0 vs. dimensionless frequency ω/ω_H under different damping ratios ξ_0 .

A UMAT program in ABAQUS is developed using Eq. (3) and Newmark's average acceleration method. The UMAT is first validated by a single-degree-of-freedom (SDF) system free vibration test where mass $m = 100$ kg, stiffness $E_0 = 40$ MPa, cross-section area $A = 10^{-4}$ m², length $l = 1$ m, displacement $u_0 = 0.015$ m, and velocity $\dot{u}_0 = 0$ m/s² at time zero. The damping ratio $\xi_0 = 0.05$. The circular frequency of the SDF system $\omega_1 = 6.32$ rad/s. Therefore, ω_H can be set to 21 rad/s and then the frequency range becomes [1.05 rad/s, 21 rad/s], which can cover ω_1 . From Appendix Table A1, $\bar{p}_1 = 15.669$, $\bar{p}_2 = 17.218$, $\bar{q}_0 = 0.800$, $\bar{q}_1 = 15.060$, and $\bar{q}_2 = 19.677$. A comparison between the finite element results and the theoretical result is shown in Fig. 1. The figure indicates that the user subroutine can precisely approximate the dynamic response of the SDF system.

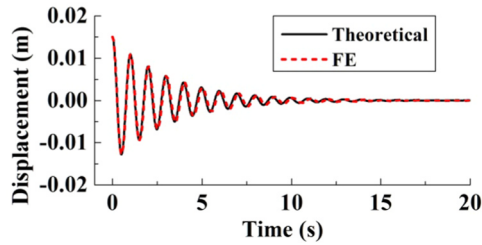


Fig. 1. Theoretical and FE free vibration curves of an SDF system.

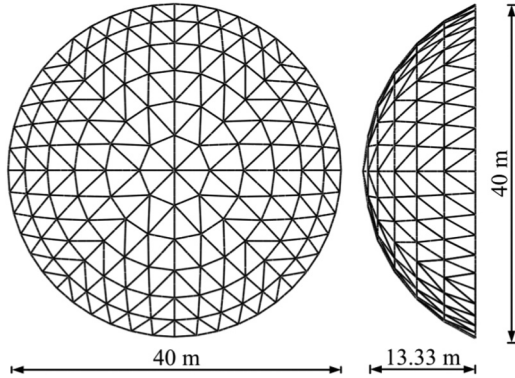


Fig. 2. Dimensions of single-layer reticulated dome.

3. Shaking table test

3.1. Description of prototype model and test dome

The prototype structure was a 40-m-span Kiewit-8 (K8) single-layer reticulated dome (Fig. 2). The basic information of the dome is listed in Table 1.

Considering the dimensions of the shaking table and fabrication conveniences, some simplifications were made to design the test model. The test model was simplified to a Kiewit-6 (K6) single-layer reticulated dome with a span of 2.7 m and rise-span ratio of 1/3. Table 2 shows a summary of the similarity ratios. Based on the similarity ratios, the dimensions of the test model were obtained as listed in Table 3. It is noted that the cross section of steel tubes in the test model was replaced by a configuration of $\varnothing 8 \text{ mm} \times 1 \text{ mm}$ because the steel tubes would be difficult to be obtained if the cross section was designed by strictly following the length similarity ratio. Spherical iron blocks were used as the middle joints, and lumped masses and the steel tubes were welded to the iron block surface. At the bottom of the test model, the steel tubes were welded to hemispheric supports fixed in the rigid steel base, which was bolted to the shaking table.

The test model incorporated 72 steel members, 19 middle joints, and 18 hemispheric supports. Fig. 3(a) shows a plan view of the test model and the numbering of the joints and supports. Fig. 3(b) shows an elevation view of the model. The shake table excitation direction is along the x-direction, and the gravity direction is along the z-direction.

Table 1

Model properties of single-layer reticulated dome.

Model Information	Value
Span (L)	40 m
Dome Height (f)	13.33 m
Rise-span ratio (f/L)	1/3
Tube sections (all members)	$\varnothing 120 \text{ mm} \times 4.5 \text{ mm}$
Geometrical imperfection	L/300
Roof loads	180 kg/m ²
Damping ratio ξ_0	0.02

Table 2

Similarity ratios (test model/prototype structure).

Variable	Value
Length (S_l)	1/15
Elastic modulus (S_E)	1.0
Stress (S_S)	1.0
Acceleration (S_a)	1.0
Time (S_t)	0.26
Density (S_ρ)	1.0
Load per area (S_q)	1.0

Table 3

Model information of test dome.

Item	Value
Span (L)	2.7 m
Height (f)	0.9 m
Rise-span ratio (f/L)	1/3
Tube sections (all members)	$\varnothing 8 \text{ mm} \times 1 \text{ mm}$
Middle joint iron block	$\varnothing 247 \text{ mm}$, 62 kg, 0.378 kg m ² (rotary inertia)
Bottom support	$\varnothing 100 \text{ mm}$ hemisphere

Fig. 3(c) shows the distributions of the middle joints and the bottom supports. Young's modulus and the yield stress of the steel tubes were $1.94 \times 10^5 \text{ MPa}$ and 340 MPa, respectively. After fabrication of the dome model (Fig. 4), the coordinates of the joints and supports were surveyed and are listed in Appendix Table A2.

3.2. Instrumentation and transducers

The displacement and acceleration transducers were mainly installed on joint numbers 0, 3, 4, 5, and 14. For convenience, these joints are denoted by A, B, C, D, and E in Fig. 5(a). Because the shake table had only one excitation direction, the LVDTs were all set along the excitation direction (x-direction) and laser sensors along the vertical direction (z-direction). Seventeen strain gauges were mainly distributed on one quarter of the dome members. Each strain gauge was attached to the middle part of the tube member. An LVDT (H6) and an accelerometer were set on the rigid steel base to acquire the actual x-direction input motion records. The distribution of the transducers is plotted in Fig. 5(b).

3.3. Test program

In order to acquire stable and sustained structural responses, a sine excitation with a frequency of 3 Hz and duration of 20 s was adopted as the input acceleration to the base of the test model. One recorded acceleration time-history curve of the base is shown in Fig. 6. Owing to some technical problems with the shaking table, the actual output sine excitation incorporated sudden impulsive accelerations that occurred at $t = 0.1 \text{ s}$ and $t = 20.1 \text{ s}$ in Fig. 6. The sequence of input motions is summarized in Table 4. Prior to every sine excitation, a white-noise excitation with a wide frequency range and low amplitude was performed to extract the natural vibration information of the test dome. The test dome model stayed in the elastic state during the entire shaking table test.

3.4. Finite element model

The technique of analytical rigid bodies in ABAQUS was employed to take into account the joint block sizes and rotation effects. The rigid bodies were modelled in the actual sizes and surveyed locations. The ball centres of the rigid bodies were assigned a mass of 64 kg and rotary inertia of 0.378 kg m². Considering the fabrication deviation of the steel tube, the positions of the steel tube ends were assumed at 10 mm in

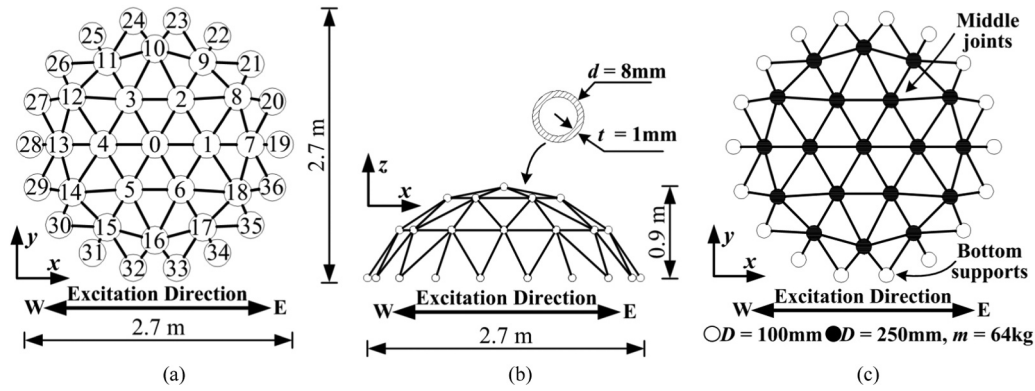


Fig. 3. Test dome dimensions and joint numbering and information.

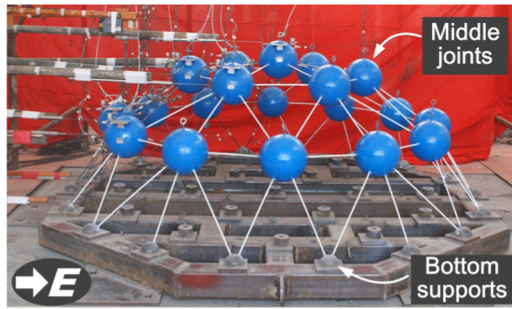


Fig. 4. View of test dome on shaking table.

distance from their ideal coordinates randomly, but they were still on the surfaces of the ball joints. The steel tube ends were tied to the corresponding surfaces of the rigid bodies.

Five B31 elements of eight cross-sectional material integration points each were used to model every steel tube member. Because the dome remained in elastic status during all sine excitations, the steel tube members were assigned only elastic properties. Young's modulus and Poisson's ratio of the tube members were set to 1.94×10^5 MPa and 0.3, respectively. Because the modal damping ratios of the test dome were around 0.0025, the parameters for the CD model were $\bar{p}_1 = 16.778$, $\bar{p}_2 = 20.287$, $\bar{q}_0 = 0.989$, $\bar{q}_1 = 16.744$, and $\bar{q}_2 = 20.421$ according to Appendix Table A1. ω_H was set to 796.5 rad/s and then the frequency range valid for the CD model was [39.8 rad/s, 796.5 rad/s]. In the RD model, $\alpha_M = 0.13968$ and $\beta_K = 3.75 \times 10^{-5}$ as calculated from the 1st and 10th vibration frequencies of the test dome.

The six degrees of freedom of all boundary nodes were all constrained.

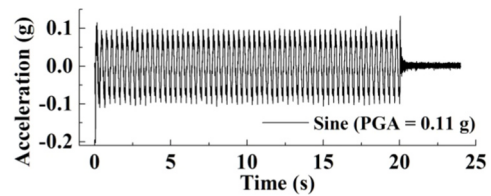


Fig. 6. Acceleration time-history curve of test dome base (PGA = 0.11 g).

Table 4

Test sequence.

Test	Motion type	Peak ground acceleration (PGA) (g)
#1	Sine	0.11
#2	Sine	0.22
#3	Sine	0.31
#4	Sine	0.44
#5	Sine	0.54
#6	Sine	0.73
#7	Sine	0.82

3.5. Natural vibration frequency analysis

Fig. 7 shows the first six vibration modes in the numerical analysis. In addition to the translational deformations, obvious rotational deformations were found in the middle joints. This agreed with the test observations. The first 10 vibration frequencies and damping ratios of the test and FE are listed in Table 5. The damping ratio and frequency of the test dome were identified by empirical-mode-decomposition-based (EMD-based) random decrement technique [27]. The detailed procedures are described in paper of He et al. [27]. The difference between the test and FE in frequency values is very small, and the maximum

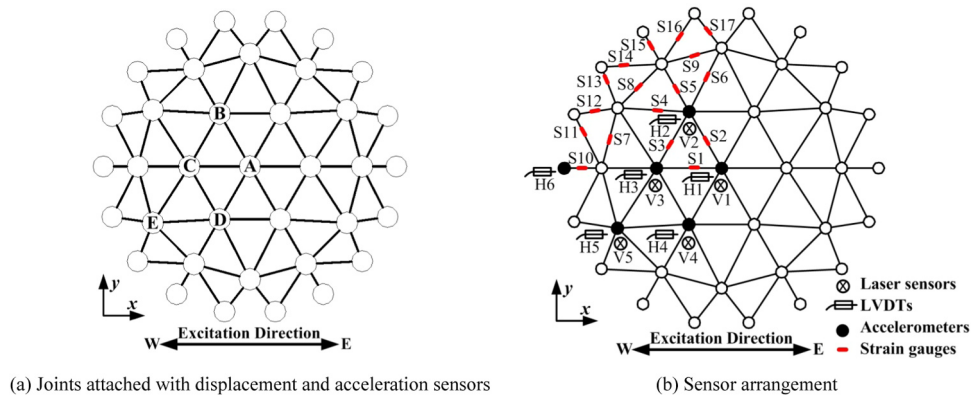


Fig. 5. Instrumentation of test dome.

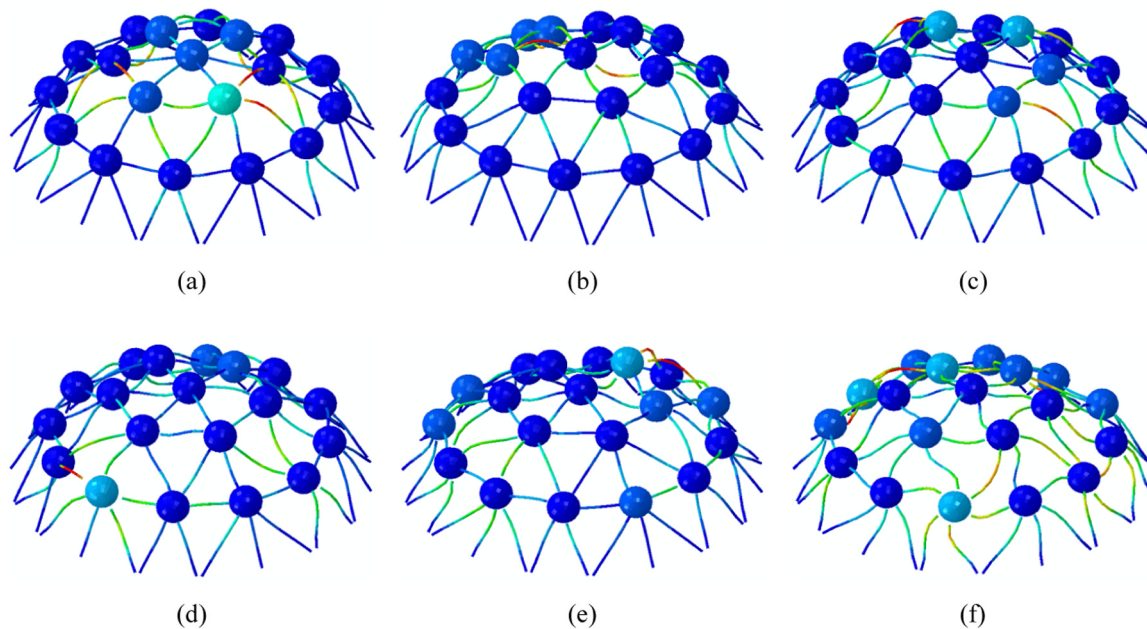


Fig. 7. First six vibration modes of FE model.

absolute error is 6.8%. This good agreement confirms that the FE model is accurate in simulating the dynamic behaviours of the test model. The identified damping ratios of different vibration modes of the test model were around 0.0025. The CD model can better evaluate the damping ratio of the first 10 vibration modes and has slightly smaller errors than the RD model.

3.6. Simulation of structural responses

Because of the middle joint rotational effect, the displacement and acceleration data (acquired by the sensors installed on the surfaces of the joints) incorporated additional values induced by corresponding rotations. The test acquired x-direction displacement and acceleration data of joint-A are equal to $u_x + 0.1235 \cdot \sqrt{2}/2 \cdot \theta_y$ and $\ddot{u}_x + 0.1235 \cdot \sqrt{2}/2 \cdot \ddot{\theta}_y$, while the test acquired data for the remaining joints (B, C, D, and E) are equal to u_x and $\ddot{u}_x$. The test acquired z-direction displacement and acceleration data of joint-B are equal to $u_z + 0.1235 \cdot \sqrt{2}/2 \cdot \theta_x$ and $\ddot{u}_z + 0.1235 \cdot \sqrt{2}/2 \cdot \ddot{\theta}_x$, while the test acquired data of the rest joints (A, C, D, and E) are equal to $u_z - 0.1235 \cdot \sqrt{2}/2 \cdot \theta_x$ and $\ddot{u}_z - 0.1235 \cdot \sqrt{2}/2 \cdot \ddot{\theta}_x$. u_x and $\ddot{u}_x$ are the x-direction displacement and acceleration of the ball centre of the corresponding joint. θ_x and $\ddot{\theta}_x$ are the x-direction rotation and angular acceleration of the ball centre, respectively, as are u_z , $\ddot{u}_z$, θ_y , and $\ddot{\theta}_y$.

Fig. 8 presents three time-history curves in the x-direction relative acceleration of joint-D under an excitation case with a PGA of 0.82 g. Except for the obvious pulsed shocks at $t = 0.1$ s and $t = 20.1$ s, the CD

model yields larger responses than the RD model, and the curve is much more consistent with the test curve in the time range [0 s, 20 s], especially with regard to the peaks of the test curves. In the free vibration phase, the CD model best approximates the test results. Fig. 9(a)–(f) present comparisons of some time-history curves of typical structural responses under an excitation of PGA 0.73 g between the test and the CD model. Good agreement is found in both the sine excitation phase and the free vibration phase.

In order to evaluate the structural dynamic responses under sine excitations, the value, which was half the sum of the maximum and minimum values of the response curve, was selected as the representative maximum structural response. The corresponding maximum and minimum values were picked from the time range [3 s, 20 s] to obtain the actual dynamic responses under sine excitations and to eliminate the influence of the shock at $t = 0.1$ s.

Fig. 10(a)–(e) present the representative maximum x-direction and z-direction relative displacements vs. PGA. The structural responses are almost proportional to the PGA, which indicates that the structure stayed in elastic status. The x-direction responses of the CD model are 5–10% larger than that of the RD model, and show good agreement with the test responses except for joint-D. The comparison of z-direction responses shows that the CD model results are 10–20% higher than those of the RD model apart from joint-C and joint-E. In addition, smaller relative differences from the test results are found in the CD model results. The z-direction results of joint-C and joint-E for the RD model are much larger than those for the CD model and test results

Table 5

Frequencies and damping ratios of test and FE.

Vibration mode	Exp. frequencies (Hz)	FE frequencies (Hz)	Error (%)	Exp. damping ratio (%)	CD model (%)	Error (%)	RD model (%)	Error (%)
#1	6.338	5.9069	−6.80	0.282	0.252	−10.64	0.250	−11.33
#2	6.739	6.4144	−4.82	0.318	0.256	−19.50	0.244	−23.17
#3	7.968	7.6574	−3.90	0.301	0.265	−11.96	0.233	−22.47
#4	8.284	8.3062	0.27	0.249	0.267	7.23	0.232	−6.92
#5	8.964	8.8428	−1.35	0.269	0.269	0.00	0.230	−14.65
#6	11.021	11.593	5.19	0.205	0.268	30.73	0.231	12.53
#7	12.546	12.309	−1.89	0.232	0.265	14.22	0.236	1.90
#8	13.049	13.794	5.71	0.229	0.264	15.28	0.239	4.33
#9	14.134	14.342	1.47	0.26	0.261	0.38	0.245	−5.71
#10	14.895	14.953	0.39	0.258	0.258	0.00	0.250	−3.06

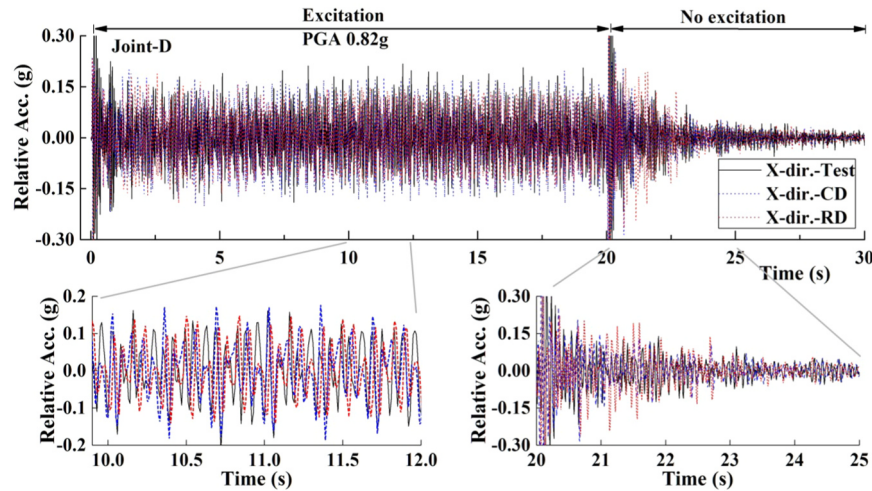


Fig. 8. X-direction relative acceleration time-history curves of joint-D under PGA = 0.82g.

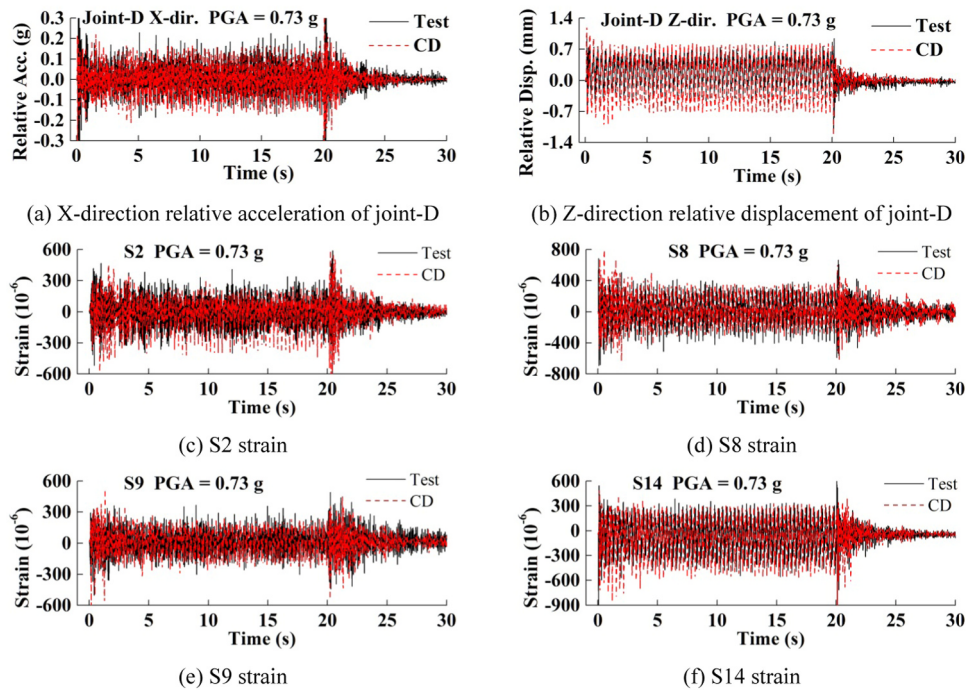


Fig. 9. Time-history curves of structural dynamic responses under PGA = 0.73g.

because the calculated rotational angles of the RD model are much higher than those of the actual conditions. Effective simulations of macro deformations of the test dome verified the accuracy of the CD model.

Fig. 11(a) presents comparisons of the tube member representative maximum strain vs. PGA. The stress results for the CD model are also at least 10% larger than those for the RD model except for S3. In the CD model, all three strain gauges (S1, S3, and S11) match the test results better than the RD model. Fig. 11 (b)–(e) shows comparisons of all strain gauge responses between the test results and CD model results. The CD model has small differences in estimating the member dynamic behaviours of the test dome model.

4. Finite element model of prototype single-layer reticulated dome

The aforementioned prototype single-layer reticulated dome was modelled in ABAQUS as in Fig. 12 to conduct incremental dynamic

analyses (IDA [28]). The joints at the bottom of the dome were all pinned as supports to the ground. The roof loads were uniformly distributed and transformed to their equivalent joint masses, modelled as MASS elements in ABAQUS. Four B31 elements with eight cross-sectional material integration points each were used to model every steel tube member. A linear kinematic hardening constitutive model (Fig. 13) was adopted to simulate the relationship between the stress and strain of the steel tube elements. Young's modulus E_0 was 2.06×10^5 MPa, Poisson's ratio was 0.3, the yield stress σ_y was 235 MPa, and the post-yielding stiffness was taken as $0.01E_0$ or 2.06×10^3 MPa.

A natural vibration frequency analysis of the dome was carried out. From Fig. 14(a), some obvious steps are observed, which means that some higher vibration frequencies are more dominant. The total effective mass of the first 200 vibration modes exceeds 75% of the total structural mass in three directions. Therefore, the first 200 vibration frequencies are sufficient to take into account the complex damping model. The first 200 vibration frequencies are shown in Fig. 14(b). The

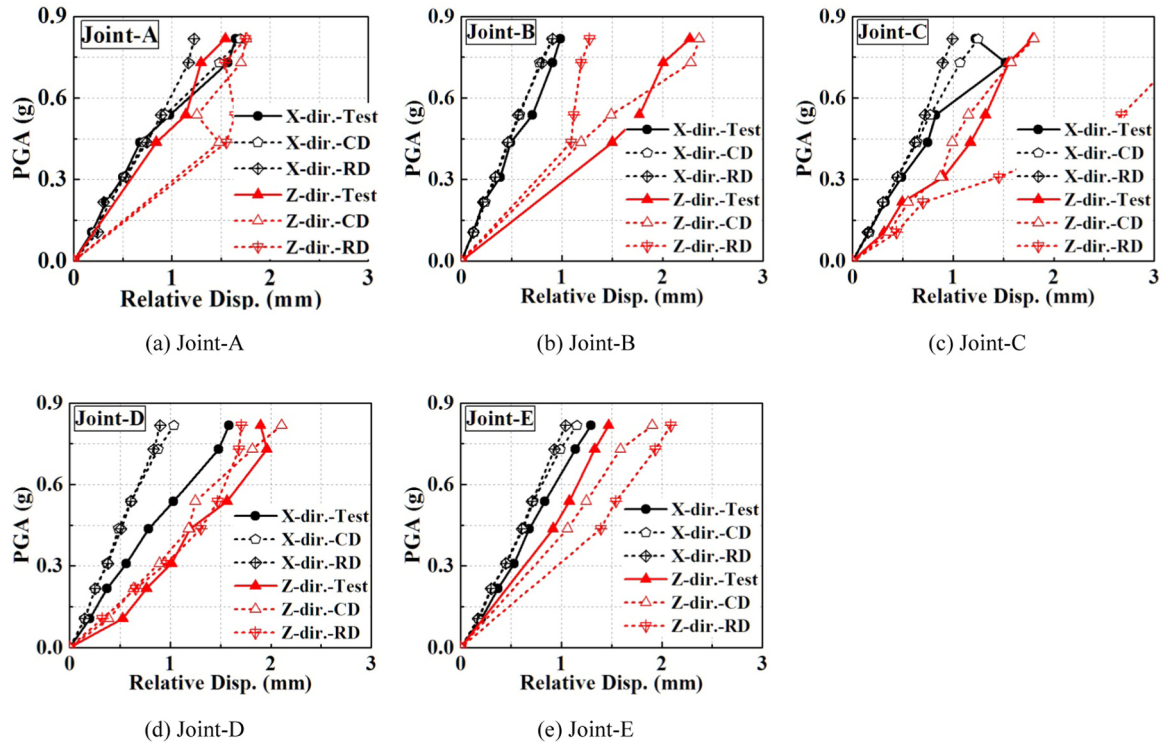


Fig. 10. Comparisons of representative maximum relative displacement vs. PGA with test, CD model, and RD model results.

1st vibration frequency is 18.94 rad/s, the 200th is 133.6 rad/s, and the ratio between the 1st and 200th is 1/7.05. This is larger than 1/20, which indicates that the five-parameter model can cover the first 200 frequencies.

According to Appendix Table A1, the parameters of the

CD model for damping ratio $\xi_0 = 0.02$ were $\bar{p}_1 = 16.200$, $\bar{p}_2 = 18.885$, $\bar{q}_0 = 0.915$, $\bar{q}_1 = 15.941$, and $\bar{q}_2 = 19.912$. ω_H was set to 330 rad/s to make the first 200 vibration frequency range of the dome included in the valid frequency range of the CD model. In the RD model, the 1st and 120th vibration frequencies (18.94 rad/s and

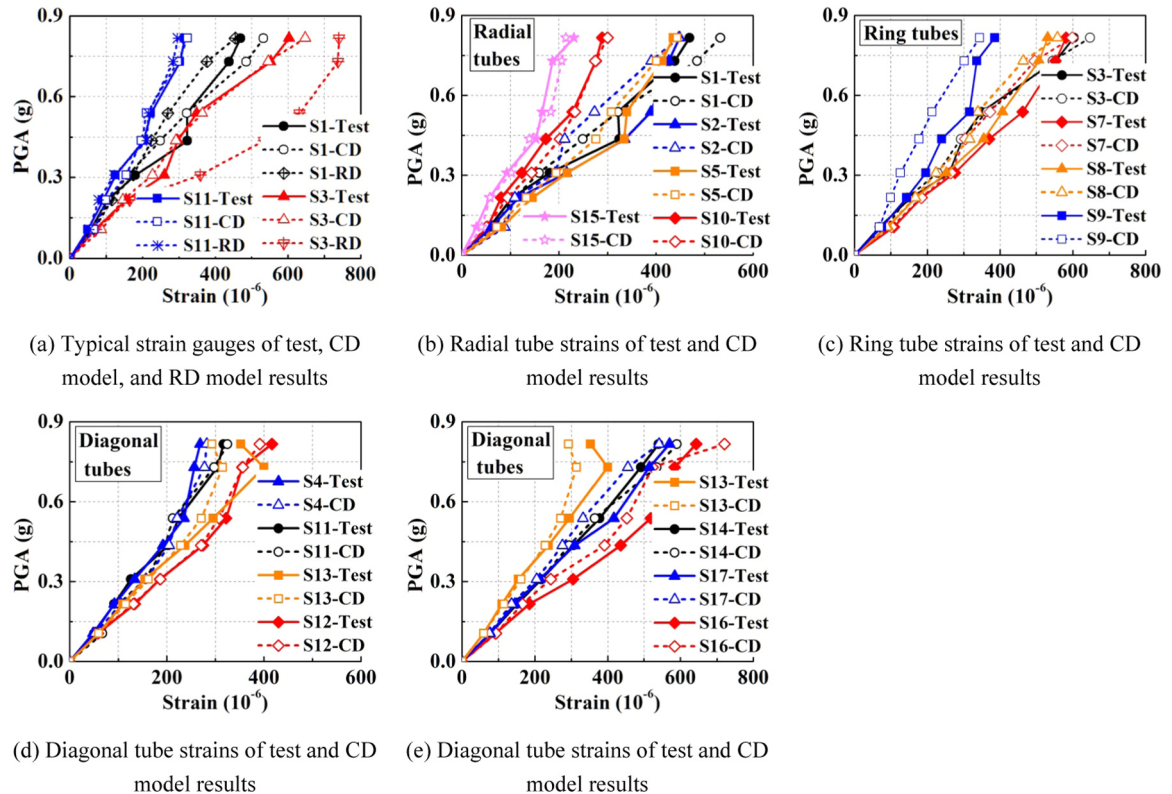


Fig. 11. Comparisons of representative maximum strain vs. PGA.

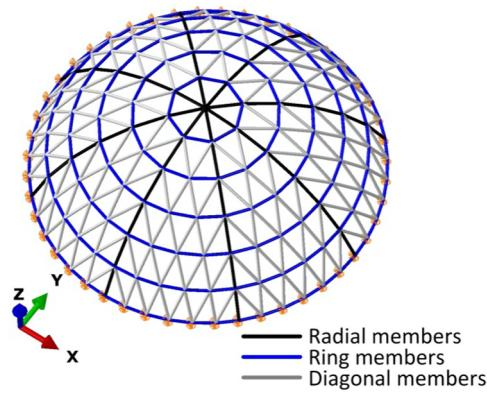


Fig. 12. Model of single-layer reticulated dome.

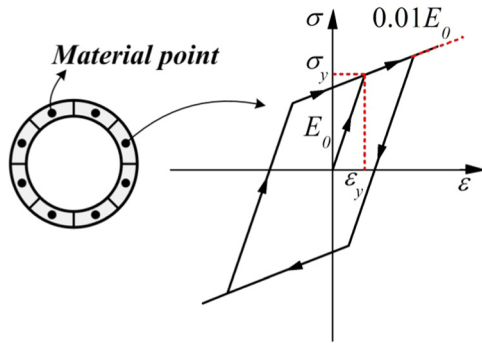


Fig. 13. Tube cross section and linear kinematic hardening model.

43.93 rad/s) were selected to determine the two coefficients α_M and β_K in Eq. 1. $\alpha_M = 0.529353$ and $\beta_K = 0.000636$. A comparison between the CD model and RD model for the first 200 vibration frequencies is shown in Fig. 15. The modal damping ratio of the CD model fluctuates slightly around the designated damping ratio 0.02, and the maximum deviation is 8% over the entire specific range. The modal damping ratio of the RD model retains in a reasonable value smaller than 0.02 within the first 120 frequencies, but it increases abruptly with vibration frequency after the 120th mode. The maximum deviation is -9%, slightly bigger than CD model, within the first 120 vibration frequencies, while the maximum error becomes 120% in the 200th mode. Therefore, the CD model is more accurate in modal damping ratio approximation than the RD model over a wide frequency range.

A set of three historic ground motions [Taft (1952), El Centro (1940), and Northridge (1994)] were selected as uniform excitations to the bottom supports of the dome. When performing the IDA, the ground motion intensity measure (IM) was set to the PGA, the damage measure (DM) was set to the maximum nodal relative deformation, and the percentage of yielded members at different levels. The symbol nP is

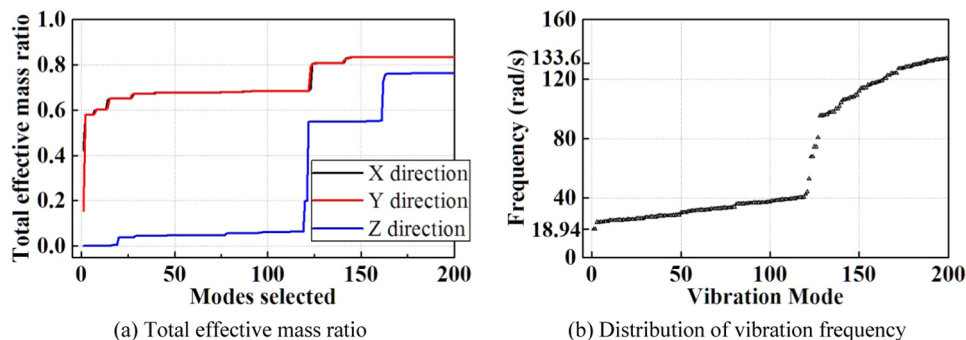


Fig. 14. Vibration frequency properties of dome.

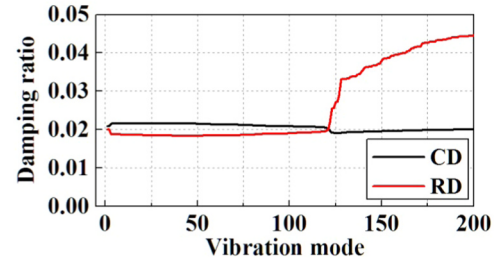


Fig. 15. Modal damping ratios calculated by CD model and RD model.

used to describe the plastic development level of the member cross section, based on the yielded number of material integration points of the cross section [3,29]. For example, 1P means that one material integration point on the cross section is yielded, and so on. The nonlinear IDA in this paper covered the elastic, plastic, and global collapse states of the reticulated dome.

5. Effects of complex damping on structural dynamic responses

The characteristic responses of the dome for the CD model and RD model were extracted and compared to investigate the influence of complex damping on the seismic responses of a single-layer reticulated dome. Fig. 16 shows the characteristic responses of the dome subjected to the three historic ground motions. From Fig. 16(a), it is noted that the ultimate seismic loads of the dome employing the CD model are all lower than those of the RD model at 10%, 4%, and 17%, respectively. Therefore, the critical failure load of the dome will decrease by an average of 10% when introducing complex damping. Fig. 16(b) shows the IDA curve with the maximum nodal relative displacement vs. the PGA level. The difference in the nodal relative displacements between the CD model and RD model varies with the ground motions. In the case of the Taft motion, maximum displacements for RD model are slightly larger than that for CD model before $PGA = 0.7$ g, but maximum displacements for RD model become smaller than that for CD model around 30% after $PGA = 0.8$ g. In the other two motions, the deformation responses of the two damping models have minor differences when the dome stays at the elastic level, but the RD model results become larger than those of the CD model after the dome becomes severe plastic. From Fig. 16(c) and (d), the ratio of yielding members in the CD model is slightly higher than that of the RD model. This means that the structural members bear higher internal forces and smaller damping forces in the CD model under the same dynamic forces.

Fig. 17(a)–(c) present comparisons of the time-history curves of nodal z-direction relative displacements under the Taft motions. Z-direction residual deformations from equilibrium positions can be seen at the ends of the motions. In general, the deformations in the CD model are larger than those in the RD model. Fig. 18(a)–(f) illustrate the distributions of yielded members to different levels and the largest deformation locations of the CD model and RD model under Taft

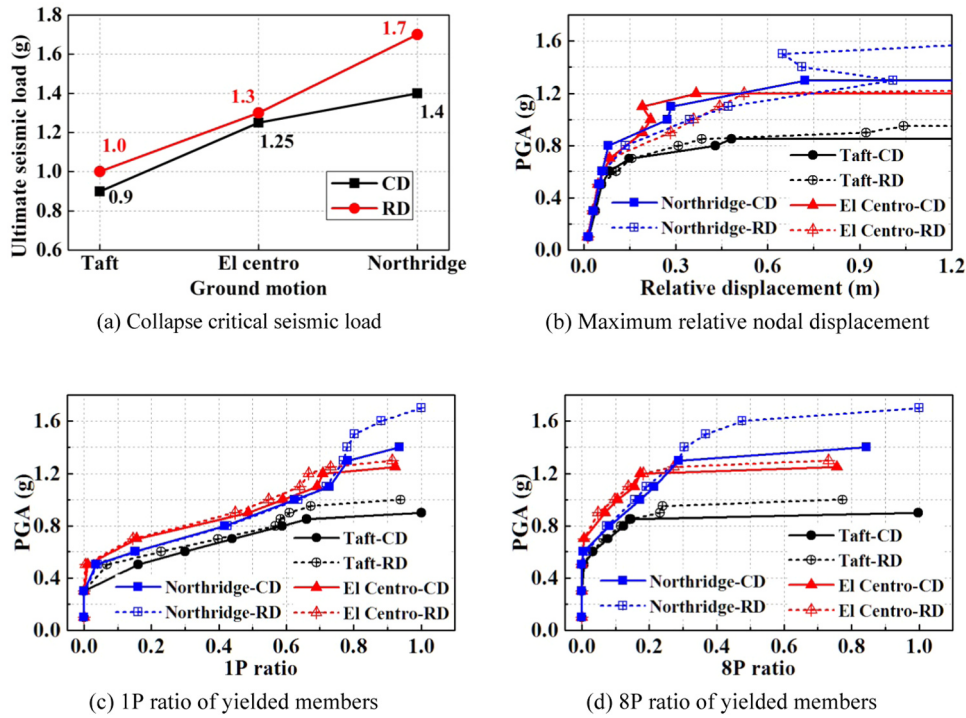


Fig. 16. IDA summaries of dome under different ground motions.

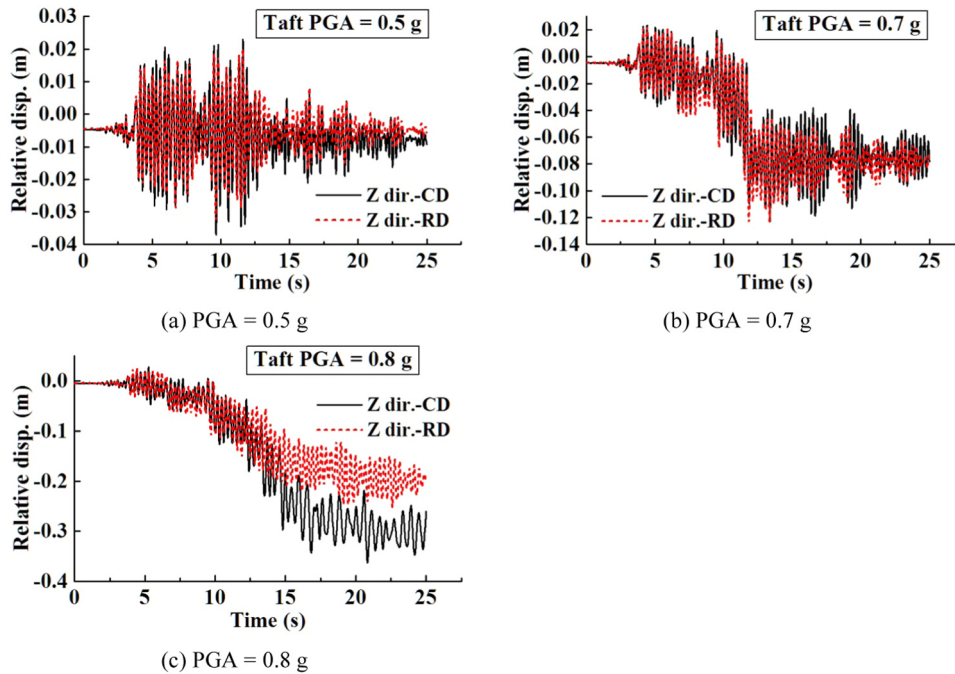


Fig. 17. Characteristic nodal relative displacement time-history curves of CD model and RD model under Taft of different PGAs.

motions. The ring members of the dome bear the largest dynamic forces and first become plastic. Much more ring members become yielded in CD model than RD model subjected to Taft motion with intensity of 0.5 g. In the case of $PGA = 0.8$ g, a large number of members develop into plasticity, and excessive local deformations are found near the

severe plastic development area, which will initiate a structural collapse under higher PGAs. The distributions of the plastic members of the two damping models are similar. However, more plastic members and severer plastic developments in top ring members are found in the CD model.

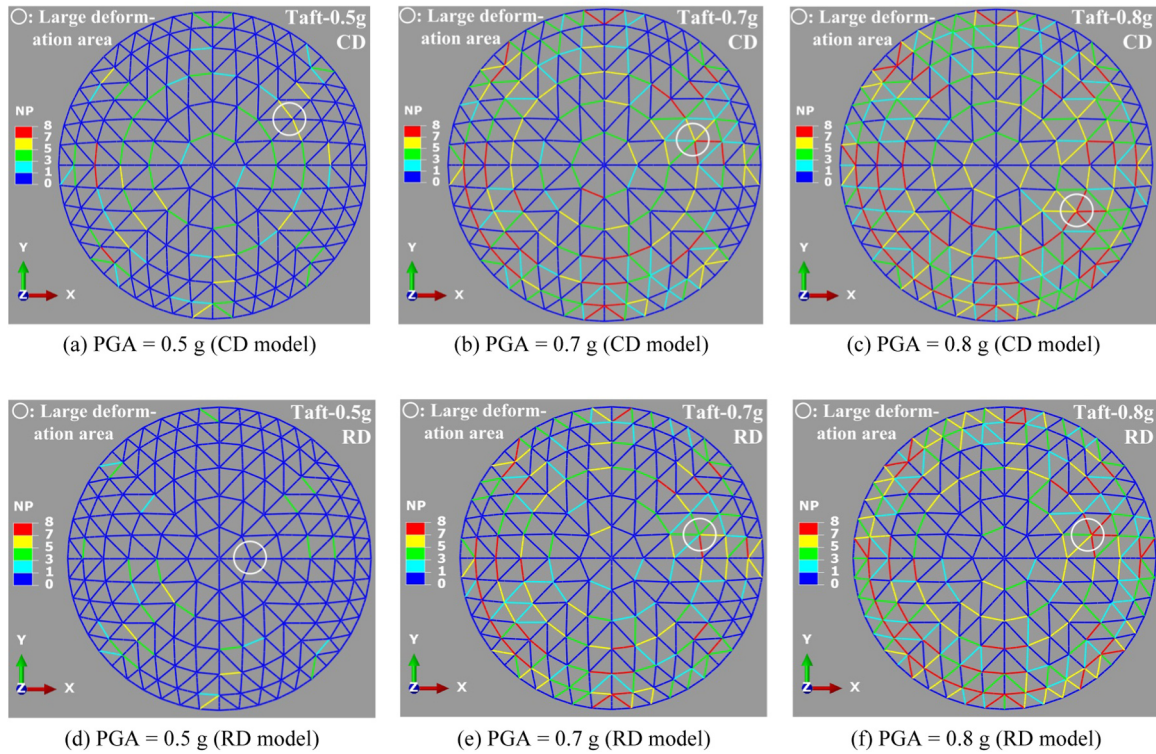


Fig. 18. Distribution of yielded tube members to different levels under Taft of different PGAs.

6. Conclusion

A five-parameter integer-order derivative model was implemented to systematically investigate the effects of complex damping on the dynamic responses of a single-layer reticulated dome subjected to earthquakes. A shaking table test of a scaled dome was also carried out to obtain detailed dynamic responses and validate the application of the complex damping model.

The developed UMAT program was verified by comprehensive comparisons of shaking table test results and CD and RD model results of a scaled single-layer reticulated dome. Better agreement with identified modal damping ratios, structural free vibration curves, and the maximum representative responses of the test results validated the proposed CD model in the numerical seismic analyses of single-layer reticulated domes.

An FE analysis of the natural vibration properties of the prototype single-layer reticulated dome showed that the complex damping simulated by the five-parameter integer-order derivative model can

maintain the damping ratio of every vibration mode that fluctuates slightly around a specific damping ratio value over a frequency range, which includes sufficient dominant vibration modes.

An IDA case study of a single-layer reticulated dome showed that the ultimate seismic load of the dome decreased by an average of 10% when taking into account the complex damping. The maximum nodal relative displacement had a slight difference when the dome was in the elastic state, but with the structural plastic development, a notable difference could be seen. The CD model increased the internal forces of the structural members and deepened the plastic development of the dome.

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Appendix

The dimensionless material constants of the five-parameter integer-order derivative model for different damping ratios are shown in Table A1. The coordinates of the joints and supports of the test dome model are listed in Table A2. Fig. A1 shows the curves of dimensionless modal damping ratio ξ/ξ_0 vs. dimensionless frequency ω/ω_H under different damping ratios ξ_0 .

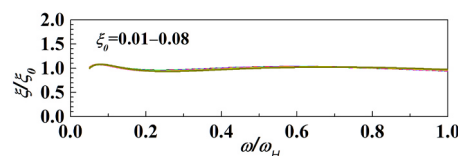


Fig. A1. Curves of dimensionless modal damping ratio ξ/ξ_0 vs. dimensionless frequency ω/ω_H under different damping ratios ξ_0 . ω/ω_H is in range [0.05, 1.0] because ξ/ξ_0 will rapidly decrease to 0.0 when ω/ω_H decreases from 0.05 to 0.0.

Table A1
Dimensionless material constants of five-parameter integer-order derivative model.

Damping ratio ξ_0	$\bar{P}_1$	$\bar{P}_2$	$\bar{q}_0$	$\bar{q}_1$	$\bar{q}_2$
0.0025	16.7781	20.2869	0.9889	16.7440	20.4213
0.01	16.5245	19.6830	0.9562	16.3906	20.2100
0.02	16.2004	18.8853	0.9145	15.9411	19.9124
0.03	15.8696	18.0547	0.8747	15.4951	19.5521
0.04	15.8680	17.8204	0.8362	15.3673	19.8210
0.05	15.6690	17.2184	0.7999	15.0602	19.6772
0.06	15.4404	16.5702	0.7653	14.7320	19.4591
0.07	14.6898	14.9213	0.7350	13.9688	18.0636
0.08	14.0358	13.5076	0.7065	13.3152	16.8758

Table A2
Coordinates of joints and supports of test dome model (unit: m).

Joint number	x	y	z
0	0.0024	−0.0053	−0.897
1	0.5593	0.0087	−0.7889
2	0.2689	0.4895	−0.7882
3	−0.284	0.4765	−0.7882
4	−0.5646	−0.0062	−0.7887
5	−0.2764	−0.4776	−0.7883
6	0.2898	−0.4781	−0.7901
7	1.0323	0.0026	−0.4744
8	0.8916	0.5153	−0.4735
9	0.5071	0.8856	−0.4744
10	−0.006	1.0311	−0.4719
11	−0.5138	0.8859	−0.474
12	−0.889	0.5124	−0.4679
13	−1.0319	−0.0028	−0.4737
14	−0.8907	−0.5182	−0.4753
15	−0.5132	−0.8952	−0.4738
16	0.0048	−1.0283	−0.4747
17	0.5195	−0.888	−0.4736
18	0.8984	−0.5078	−0.4749
19	1.3509	0.0019	−0.0059
20	1.268	0.4565	0.0016
21	1.0326	0.8666	−0.0068
22	0.671	1.1693	−0.0016
23	0.2334	1.3302	−0.0027
24	−0.2346	1.3293	−0.0043
25	−0.6783	1.1684	−0.0021
26	−1.0395	0.8616	−0.0055
27	−1.2685	0.4599	−0.0075
28	−1.3491	−0.0031	−0.0061
29	−1.2672	−0.4657	−0.0089
30	−1.032	−0.869	−0.006
31	−0.6688	−1.1674	−0.006
32	−0.2326	−1.3278	−0.0073
33	0.2319	−1.3262	−0.0057
34	0.6749	−1.1671	−0.0035
35	1.0344	−0.8675	−0.0125
36	1.2711	−0.4555	−0.0027

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